ABSTRACTS FOR THE SUMMER SCHOOL ON MAPPING CLASS GROUPS AND THEIR REPRESENTATIONS IMJ-PRG, PARIS (JUSSIEU CAMPUS), 3-7 JULY 2017

Speaker: **Tara Brendle** (Glasgow)

Title : **The integral Burau representation of braid groups**

Abstract: The Burau representation plays a key role in the classical theory of braid groups. When we let the complex parameter t take the value -1, we obtain a symplectic representation of the braid group, known as the integral Burau representation. In this talk we will give a survey of recent work with Margalit and others on braid congruence subgroups, that is, on the preimages in the braid group of the principal congruence subgroups of the symplectic groups. This survey will include a discussion of the kernel of the integral Burau representation itself, as well as connections with a wide variety of areas, including algebraic geometry and number theory.

Speaker: **François Costantino** (Toulouse)

Title : **Quantum representations of mapping class groups**

Abstract : The goal of this mini-cours is to provide a "hands on" approach to SU(2)-quantum representations of mapping class groups (the most famous example of quantum representations) in order to allow non-experts to compute matrices associated to mapping class groups elements. We will start by discussing what a TQFT is, why these objects are relevant and providing some methods to construct it (in particular the universal construction). Then we will define skein modules of 3-manifolds and discuss some of their properties with the goal of defining quantum spin networks and provide some of their basic computational properties. We will then apply the universal construction to build the SU(2)-TQFTs and provide bases and examples of computation of matrices representing mapping class group elements. We will conclude by discussing some of the properties of these TQFTs and, if time permits, we will review some of the more recent evolutions in the subject.

Speaker: **Asaf Hadari** (Hawaii)

Title: **Homological representations and individual elements of mapping class groups**

Abstract: We will discuss a recent theorem that shows that the finite dimensional linear representation theory of mapping class groups "knows" whether or not any particular element has infinite order. More specifically, given an element f of a mapping class group, we will explain how to construct a representation of the whole group where the image of f has infinite order.

Lecture 1: *Introduction.*
Abstract: We will discuss mapping class groups, automorphism groups of free groups, and their homological representations. We will also discuss the various objects that can be associated to pseudo-Anosov elements of a mapping class group.

Lecture 2: *The Magnus representation and anchored matrices.*

Abstract: We will discuss a particular (inifinite dimensional) representation of the mapping class group, and show how under certain conditions (what we will call - the anchored image condition) it can be used to provide the information we want about finite dimensional representations.

Lecture 3: *Homological shadows and vertex subgraphs*

Abstract: We will discuss the notion of a homological shadow of a pseudo Anosov map, which is a hybrid geometric/homological object associated to a pseudo-Anosov map, and discuss its relation to the anchored image condition. We will also discuss vertex subgraphs, an aspect of homological shadows that will be central for our proof.

Lecture 4: *Stabilizing vertex subgraphs.*

Abstract: We will discuss how to complete the proof, using a process called vertex subgraph stabilization.

Speaker: **Alex Lubotzky** (Hebrew University)

Title: **Arithmetic quotients of the mapping class group**

Abstract: Let $M = M(g)$ be the mapping class group of a surface of genus $g > 1$ (resp. $M = \mathrm{Aut}(F_g)$ the automorphism group of the Free group on g generators). As it is well known, M is mapped onto the symplectic group $\mathrm{Sp}(2g, \mathbb{Z})$ (resp. the general linear group $\mathrm{GL}(g, \mathbb{Z})$). We will show that this is only a first case in a series: in fact, for every pair (S, r) when S is a finite group with less than g generators and r is a $\mathbb{Q}$-irreducible representation of S, we associate an arithmetic group which is then shown to be a virtual quotient of M. The case when S is the trivial group gives the above $\mathrm{Sp}(2g, \mathbb{Z})$ (resp. $\mathrm{GL}(g, \mathbb{Z})$) but many new quotients are obtained. For example it is used to show that $M(2)$ (resp. $\mathrm{Aut}(F_3)$) is virtually mapped onto a non-abelian free group. Another application is an answer to a question of Kowalski: generic elements in the Torelli groups are hyperbolic and fully irreducible. Based on works with Fritz Gruenwald, and with Grunewald , Michael Larsen and Justin Malestein .

Speaker: **Andy Putman** (Notre Dame)

Title: **On the virtual first Betti number of the mapping class group**

Abstract: A fundamental open question about the mapping class group is whether or not its virtual first Betti number is positive. In other words, does it have a finite-index subgroup that surjects onto $\mathbb{Z}$? I will discuss a variety of tools that have been developed to address this question, with a focus on the "higher Prym representations" that I introduced with Ben Wieland. Very little background about the mapping class group will be assumed.

Speaker: **Alan W. Reid** (Rice)

Title: **Subgroups and quotients of Mapping Class Groups via TQFT representations**

Abstract: In this talk we describe joint work with G. Masbaum on some applications of the projective unitary representations arising in Topological Quantum Field Theory towards understanding quotients of Mapping Class Groups (finite and infinite), as well as the subgroup stucture of Mapping Class Groups (e.g. a question of Ivanov about a variant of the Frattini subgroup).

Speaker: **Ramanujan Santharoubane** (Virginia)

Title: **Surface groups representations and Homology of finite covers via TQFT**

Abstract: The Witten Reshetikhin-Turaev TQFT produces finite dimensional representations of mapping groups of surfaces. Using the Birman exact sequence, we will see how these representations induce representations of surface groups. These TQFT representations of surface groups have the property that every simple loop acts with finite order and they have infinite image. Moreover they give an example of points on character varieties fixed under the action of the mapping class groups but with infinite images. Finally, I will show how we can apply integral TQFT (introduced by Gilmer and Masbaum) techniques to these TQFT representations in order to build finite regular covers of surfaces. These finite covers have the property that their integral homology is not generated by pullbacks of simple closed curves on the base.

This talk represents a joint work with Thomas Koberda.

Speaker: **T. N. Venkataramana** (TIFR, Mumbai)

Title: **The Burau representation and monodromy groups**

Abstract: The braid group on n letters is the mapping class group of the complex plane minus n punctures. The Burau representation of the braid group is closely related to the monodromy representation of a special family of cyclic coverings of the projective line ramified only at n points (sometimes n+1). In this course, we sketch a proof that the monodromy group is an arithmetic group provided the degree of the cyclic covering is smaller than n.

The proof involves certain results on arithmetic groups which use methods from the proof of the congruence subgroup property, and we review these results as well.

We also sketch the relation of this monodromy group with the monodromy of the "Lauricella hypergeometric functions", and briefly review the related work of Deligne and Mostow.